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Experimental observation of 'pure helical phase' interference using moiré fringes generated from holograms with dislocations

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Abstract
A Laguerre–Gaussian (LG) beam possesses a uniform orbital angular momentum of $l\hbar$ due to its helical phase term $\exp(-\mathrm{i}l\phi)$, where l is the topological charge of the beam. The helical phase or the topological charge of any LG beam can be observed by causing it to interfere with a Gaussian beam or its mirror image. In this paper we propose a novel method of using moiré fringes to study the changes in the helical phase and the topological charge upon interference between any two LG beams of any arbitrary helicity. Using this method we are able to observe the interference purely between the helical phases which can be readily observed using white light illumination as shown in our experimental results. Such pure helical phase interference is not readily observed in conventional interferometry.

Keywords: computer generated holograms, interference, holographic interferometry

1. Introduction

Computer generated holograms with dislocations have been widely employed by researchers to generate optical vortex or Laguerre–Gaussian (LG) beams [1, 3–5, 8]. The optical vortex has been used in the study of nonlinear optics [2], optical rotation [3] and quantum entanglement [4]. The production of holograms with dislocations is based on the fact that when a plane wave interferes at an angle with a LG beam, a dislocation intensity pattern is formed. The LG full mode in cylindrical coordinates is given by equation (1):

$$E(\mathrm{LG}_{\mathrm{p}}^{l}) \propto \left[\frac{r\sqrt{2}}{w(z)}\right]^{|l|} L_{\mathrm{p}}^{|l|}\left[\frac{2r^2}{w^2(z)}\right]\exp\left(\frac{-r^2}{w^2(z)}\right) \times \exp\left(\frac{-\mathrm{i}kr^2z}{2(z^2+z_{\mathrm{R}}^2)}\right)\exp(-\mathrm{i}l\phi) \times \exp\left(-\mathrm{i}(2p+|l|+1)\tan^{-1}\frac{z}{z_{\mathrm{R}}}\right) \quad (1)$$

where $L_{\mathrm{p}}^{|l|}$ is a generalized Laguerre polynomial, $w(z)$ is the radius of the beam at the position z, z_{R} is the Rayleigh range.

In recent years, researchers have used these computer generated holograms with dislocations to intensively study the topological charge dynamics of vortices nested within a single Gaussian beam [5] and interference effects between the LG beams of the same topological charge [6]. In most of the studies done, a plane wave from a coherent source is directed onto the computer generated hologram to generate the LG beam. The LG beam generated is then caused to interfere with a plane wave [1] or another LG beam with a different topological charge [7, 8]. The resulting interference pattern will provide the information of the helical phase structure of the original LG beam as well as the helical phase structure of the other LG beam.

There have been few reports using such an interferometry technique on the topological charge [8] and orbital angular momentum (OAM) interaction [5] when two LG beams of different charge and OAM made experimentally to interfere.

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One possible reason is the difficulty of obtaining a simple and flexible optical interferometer set-up for experimentally observing the interference between LG beams of any arbitrary helical phase structure.

In this paper we report that the moiré fringe pattern generated when two computer generated holograms with dislocations of any topological charge are superimposed are identical to the fringe pattern when the 'pure helical phases' of any two LG beams are made to interfere. In other words, the moiré fringe observed is the direct representation of the helical phase interaction between the two LG beams. This method can also be expanded and used to observe the interference among other beams with helical phase structure of $\exp(-\mathrm{i}l\phi)$. The computer generated holograms can be easily made from photographic slides or transparencies.

2. The mathematical similarity of pure helical phase interference with moiré fringes generated from superimposed holograms with dislocations

In conventional interferometry, the interference pattern provides information on the phase difference between the two interfering coherent beams. In the moiré fringe phenomenon, the observed moiré patterns provide information on how the individual helical phase changes upon interference.

First, we shown the mathematical similarity between the pure helical phase interference and the moiré fringe generated from superimposing holograms with dislocations.

The helical phase term of an LG beam, $\exp(-\mathrm{i}l\phi)$, is evidence of a well-defined OAM of $l\hbar$ present within the beam [9, 10], as shown in equation (2). To mathematically observe the interference of just the helical phase between two LG beams, we use the interference equation given as equation (3). I_T will show the resulting pure phase interference as an intensity pattern:

$$E(\mathrm{LG}_\mathrm{p}^l) \propto \exp(-\mathrm{i}l\phi) \quad (2)$$

$$I_\mathrm{T} = |\exp(-\mathrm{i}l_1\phi) + \exp(-\mathrm{i}l_2\phi)|^2 \quad (3)$$

where $\phi = \arctan(y/x)$.

In figures 1(A) and (B), the helical phase ramps of LG beams of azimuthal numbers $l = 2$ and -3 are shown, respectively. Figures 1(C) and (D) demonstrate the collinear phase interference between helical phases of $l = 2$ and 3 and between helical phases of $l = -2$ and 3, respectively.

Figures 1(E) and (F) present the helical phases of $l = 2$ and 3 and $l = -2$ and 3 interfering collinearly but separated at a distance of $a/20$, where a is the beam waist of the beam.

Figures 1(G) and (H) show the misaligned phase interferences between helical phases of $l = 2$ and 3, and of $l = -2$ and 3, respectively. Lastly, figures 1(I) and (J) show the misaligned phase interferences between the helical phases of $l = 2$ and 3 and of $l = -2$ and 3, at a small separation distance of $a/20$.

These helical phase interferences are mainly illustrated by equation (3), which leads to the expression $1 + \cos(l_1 - l_2)\phi$ upon simplification. Hence, the intensity pattern of the helical phase interference is directly modulated by $\cos(l_1 - l_2)\phi$. The resulting helical phase will be the azimuthal term of the cosine function which is $(l_1 - l_2)\phi$, as shown in figure 1.

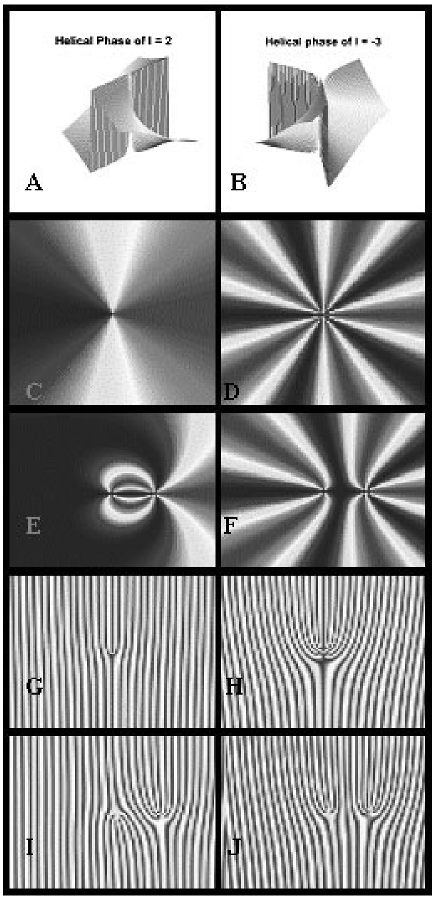

Figure 1. Shows the difference in phase interference between LG beams of $l = 2$ and 3.

Such a helical phase interference can be easily visualized and observed using the moiré fringes that arise from two holograms with dislocations under illumination with incoherent or white light.

Mathematically, computer generated holograms with dislocations can be simply expressed as $\cos(kx - l\phi)$, where k is the grating vector. When two of these holograms are superimposed onto each other, the effect will be similar to that of multiplying two $\cos(kx - l\phi)$ expressions.

Therefore, assuming that T_m is the resulting intensity distribution and T_1 and T_2 are the two expressions that represent the two gratings containing the dislocations of different charge, the moiré fringe pattern can be expressed as

$$\begin{aligned} T_m(x, y) &= T_1(x, y) \times T_2(x, y) \\ &= \cos(kx - l_1\phi) \times \cos(kx - l_2\phi) \\ &= \tfrac{1}{2}[\cos(l_1 - l_2)\phi + \cos(2kx - (l_1 + l_2))\phi], \end{aligned} \quad (4)$$

where $\phi = \arctan(y/x)$.

From equation (4) we can see that the first term of the expression is the same as that of the helical phase interference $\cos(l_1 - l_2)\phi$, while the second term shows the grating distribution about the x-axis (vertical lines perpendicular to the x-axis). Hence, when the two holograms overlap, we would expect to see helical phase interference. However since the dislocation holograms consist of vertical line gratings, the phase interference will be displayed in the form of moiré fringes. In order to verify such a deduction, we calculate the

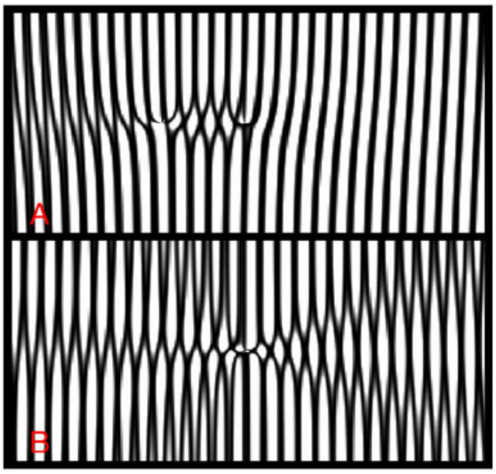

Figure 2. (A) and (B) show the numerically calculated moiré fringes for two holograms of dislocations $l = 2$ and 3 and $l = -2$ and 3, respectively. In (A) the holograms are shifted by five grating spacings.

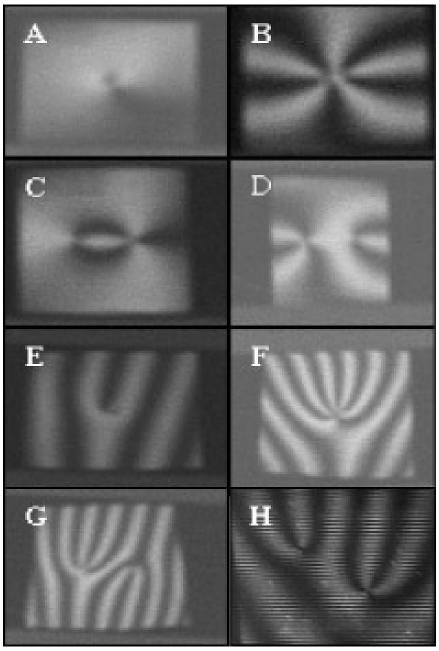

Figure 3. (A)–(H) show the actual moiré fringes for two holograms of dislocations $l = 2$ and 3 overlapping under incoherent illumination.

mathematical expression, as given in equation (4), numerically. In the numerical verification, we chose numerical values of $l_1 = 2$ or -2 and $l_2 = 3$.

In figures 2(A) and (B), we can observe that the moiré fringes are similar to the phase interference shown in figures 1(E) and (D), respectively. In figure 2(A) we substitute $l_1 = 2$ and $l_2 = 3$ into equation (4) and in figure 2(B) we substitute $l_1 = -2$ and $l_2 = 3$ into equation (4). This proves that the moiré fringes generated from the overlapping of two holograms of different topological charges are the same as the helical phase interference between the two LG beams.

3. Experimental observation of the moiré fringes

The next step is to observe the moiré fringe experimentally. Hence, we developed a set of holograms with dislocations on 35 mm photographic slides. In figure 3 we observe the expected moiré fringes when these holograms with dislocations overlap under illumination with incoherent (white) light.

Comparing these moiré fringes with the phase interference shown in figure 1, it is obvious that the phase interference of LG beams has been observed physically through the moiré fringes under the incoherent illumination.

From both theoretical and experimental results shown in figures 1 and 3, we readily observed that the resulting helical phase after collinear interference resolves into $|l_1 \mp l_2|$. This can be seen, as when the helical phase of $l = -2$ interferes with $l = 3$, the remaining helical phase is equal to $|l_1 \mp l_2| = 5$ seen in figures 1(D), (F), 3(B) and (D). As for the helical phase of $l = 2$ and 3, the resulting helical phase is equal to 1, as seen in figures 1(C), (E), 3(A) and (C).

Furthermore, it is observed that when two helical phases of $l = 2$ and 3 interfere at an angle and a small separation, the topological charge of $l = 2$ will be inverted with respect to the other. Once they approach each other, the remaining topological charge will be $|l_1 - l_2| = 1$, as seen in figures 1(G), (I), 3(E) and (G). For two helical phases of $l = -2$ and 3, the negative topological charge of -2 will appear upright with respect to $l = 3$. The remaining topological charge from them will be $|l_1 \mp l_2| = 5$, as seen in figures 1(H), (J), 3(F) and (H).

It appears that when two helicity phases of any order interfere, one of the helicity phases will have its topological charge sign reversed, as shown in figures 1(G)–(J) and figures 3(F)–(H), in simulations and experiments respectively. This will result in either the addition or the subtraction of the spiral or fork fringes.

4. Conclusions

In conclusion we have proposed and demonstrated, both theoretically and experimentally, that helical phase interference of any order of helicity can be readily observed using the moiré fringes from superimposed holograms with dislocations under white light illumination. In this method we can easily observe such pure helical phase interference without using a conventional interferometer where the wavefront curvature and the Guoy phase of the interfering laser beam have to be matched or mismatched [1].

Furthermore it is worth noting that in a conventional interferometer, the pattern of interference between the LG beams reveals the helical phase in the form of an intensity pattern only when the two interfering LG beams are from the same coherent laser source. However, in the moiré fringe analysis, there is no need for any coherent laser source to be used. All that is required is to make just two computer generated holograms with dislocations overlap and then to observe them under white light illumination.

Beijersbergen *et al* [10] have shown that an OAM of $l\hbar$ per photon is correct for all beams with the azimuthal phase structure of $\exp(-il\phi)$. This in turn implies that the OAM is solely dependent on the azimuthal phase structure [10].

Therefore, the moiré fringe phenomenon allows us to observe clearly the changes that are happening in the azimuthal phase structure of the beams. These changes in the azimuthal phase structure could in turn affect the OAM of the individual interfering light beams during interference.

Therefore, using this method, it will be much easier to observe the phase structure changes occurring between any optical fields with the azimuthal phase structure of $\exp(-il\phi)$ [11], without the need to work on any simulation programs or interferometer experiment.

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